Rule-based Tensor Mutations Embedded within LLMs for Low-Cost Mathematical Computation

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***Abstract*—Large Language Models (LLMs) have demonstrated remarkable proficiency in natural language tasks but remain inefficient and error-prone when performing deterministic mathematical computations. Existing approaches to improving mathematical reasoning rely on external symbolic engines or extensive fine-tuning on mathematical corpora, both of which introduce latency and scalability challenges. This paper proposes a novel architectural enhancement for transformer-based LLMs: the embedding of deterministic, rule-based tensor mutations directly within the model's internal computational graph. By implementing fixed-index tensor operations—such as arithmetic functions, binary operations, and matrix computations—within the embedding space of the Llama 3 3B model, we enable low-latency mathematical reasoning without modifying the core probabilistic architecture. The proposed system leverages deterministic computation pathways optimized for GPU tensor cores, significantly reducing inference latency and improving mathematical accuracy on arithmetic and linear algebra tasks.**

***Index Terms*—Multi-Layer Perceptron (MLP), Rule-based Mutation, Neural Network Architecture, Language Models, LLaMA, Long-Horizon Reasoning, Step-wise Accuracy, Model Generalization, Deep Learning, Artificial Intelligence, Training Efficiency, Inference Optimization, Neural Computation, Architecture Search, Mutated MLPs, Model Scaling, Structural Inductive Bias, Token-wise Evaluation, Parametric Efficiency, High-Performance Computing, Transformer Models, Cognitive Tasks, Reasoning Benchmarking, Neuro-Symbolic Integration.**

I. Introduction

Large Language Models (LLMs) have rapidly advanced the field of natural language processing (NLP), achieving unprecedented success across tasks such as text generation, summarization, translation, and conversational reasoning. These models, built upon transformer architectures, learn statistical patterns in tokenized language data through extensive pretraining on vast corpora. However, despite their proficiency in language understanding, LLMs consistently underperform on tasks that require deterministic mathematical computation [1] [2]. This limitation stems from the fundamentally probabilistic nature of neural network inference, which excels at pattern recognition but lacks the precise symbolic manipulation capabilities required for accurate mathematical reasoning.

Current approaches to improving the mathematical competence of LLMs follow two main paradigms. The first involves fine-tuning models on specialized mathematical datasets [3], such as arithmetic sequences, calculus problems, or algebraic equations. While fine-tuning improves performance on familiar problems, it is both computationally expensive and brittle when generalizing to unseen operations or data distributions. The second paradigm leverages Retrieval-Augmented Generation (RAG) pipelines that offload computation to external symbolic engines such as Wolfram Alpha. Though effective in some contexts, these solutions introduce substantial inference latency due to the need for external API calls and often compromise the seamless, end-to-end nature of LLM inference pipelines.

TABLE I
Comparison of LLM Computational Requirements

Model Name	Compute (PF-days)	Inference (ms/tkn.)	VRAM (GB)
GPT-2	5.6	12	3
GPT-3	3,640	75	350
LLaMA-2-7B	184	18	14
LLaMA-2-13B	368	32	26
LLaMA-2-70B	1,720	145	140
Claude 2	N/A	82	$\sim$200
GPT-4	$\sim$25,000	210	$\sim$3,000

Note— Training compute is measured in petaflop-days. Inference time is reported per token on an A100 GPU. Memory usage denotes peak VRAM during inference. Proprietary model figures are estimates.

Moreover, scaling LLMs to address such shortcomings faces practical limitations. Empirical scaling laws [4] demonstrate that beyond a certain point, increasing the number of model parameters yields diminishing returns in accuracy relative to computational cost. This is particularly evident in mathematical reasoning benchmarks, where larger models show sub-linear performance improvements despite exponential increases in compute and memory consumption. As Table I illustrates, state-of-the-art models such as GPT-4 and Claude 2 require thousands of petaflop-days of compute and terabytes of memory, yet they still fail to achieve high accuracy on elementary arithmetic problems without external assistance.

This paper addresses this gap by proposing a fundamentally different approach: embedding deterministic, rule-based tensor mutations directly within the neural network's computational graph. Instead of relying solely on statistical learning, this method introduces explicit, hard-coded mathematical operations into specific locations of the model's embedding space. By leveraging the high parallelism of modern GPUs, particularly tensor core architectures optimized for Single Instruction, Multiple Data (SIMD) workloads, these operations execute with minimal latency and no dependence on external inference pathways.

The proposed system modifies the Llama 3 3B model, an open-weight transformer, to include fixed-index mathematical functions such as arithmetic addition, matrix multiplication, and binary bitwise operations. These rule-based pathways operate deterministically on predefined sections of the token embedding space and coexist with the model's standard stochastic transformer layers. This hybrid architecture preserves the language modeling strengths of the transformer while enabling precise mathematical reasoning without additional fine-tuning or inference-time API calls.

This work contributes to the broader discourse on integrating symbolic computation into neural architectures. Prior efforts in neural-symbolic computing have explored symbolic regression, logic programming over neural graphs, and reinforcement learning for tool use [**?**]. Unlike these approaches, our method does not require training the model to learn mathematical operations; instead, it injects these operations at runtime within the forward pass of inference. This design minimizes the computational overhead associated with training while maximizing inference-time efficiency.

II. Related Works

Mathematical reasoning in artificial intelligence is broadly categorized into two complementary paradigms: *symbolic computation* and *statistical pattern learning*. Symbolic computation refers to the manipulation of mathematical objects using discrete logic, such as arithmetic operations, algebraic simplifications, or equation solving. These processes are deterministic, meaning that given the same inputs, they yield the same outputs independent of statistical variation. In contrast, statistical pattern learning, as embodied by neural networks, involves learning probabilistic relationships between tokens or symbols through exposure to large datasets. While statistical learning captures distributional patterns across language, it does not inherently encode the rules of mathematics that govern the manipulation of numbers and expressions.

Historically, symbolic artificial intelligence systems such as theorem provers, expert systems, and computer algebra systems (e.g., Mathematica, SymPy) have excelled at mathematical reasoning due to their reliance on explicit rule sets and logic engines. These systems require handcrafted rules but offer precise, explainable solutions. Neural networks, including modern large language models, learn representations of symbols as continuous vectors in high-dimensional spaces, enabling them to generate coherent text and recognize syntactic patterns. However, without explicit rules or external reasoning engines, their mathematical capabilities remain fragile and reliant on memorized patterns rather than systematic reasoning. Bridging the gap between these paradigms has become a critical area of research in neural-symbolic computing.

Efforts to improve mathematical competence in language models generally fall into three categories. The first is *data-centric approaches*, where models are fine-tuned on curated datasets containing mathematical problems, equation patterns, and arithmetic exercises. While this improves recall of memorized problem structures, it does not enable novel symbolic manipulation. The second is *tool-augmented inference*, where models are coupled with external symbolic engines like Wolfram Alpha or SymPy at runtime. These tools enable accurate computation but introduce latency, architectural complexity, and reliance on external dependencies. The third is *architectural modification*, where symbolic components are embedded directly into the model's computational graph. This approach aims to enable the model to compute symbolically during inference, preserving end-to-end differentiability and eliminating external dependencies.

Several conventions have emerged in the study of neural mathematical reasoning. Researchers distinguish between *in-context learning* of symbolic patterns (where a model memorizes examples during pretraining), *emergent reasoning* (where generalization arises without explicit training on mathematical tasks), and *symbolic execution*, where operations follow deterministic pathways independent of model weights. Additionally, evaluations often distinguish between *single-step* arithmetic, such as evaluating "3 + 5," and *multi-step* problems, such as solving algebraic expressions or nested equations. Performance on benchmarks like MATH [5] and GSM8K has revealed that while LLMs handle natural language problem descriptions well, they frequently err in the computation stage, demonstrating their probabilistic nature.

Thus, the challenge is not simply a matter of increasing dataset size or model parameters but rethinking how computation is performed within neural networks. Approaches like program synthesis, intermediate variable reasoning, and explicit mathematical instruction tuning have made progress but remain constrained by the probabilistic nature of neural inference. Embedding deterministic operations directly into the model's inference pathways represents a fundamentally different approach. Instead of predicting the answer token by token, the model can deterministically compute intermediate results within its tensor operations. This paper contributes to this emerging direction by proposing a mechanism for rule-based tensor mutations applied at specific locations within a transformer's multi-layer perceptron (MLP) sub-blocks, enabling precise symbolic computation without external tools or fine-tuning.

The gap between probabilistic language modeling and deterministic symbolic reasoning has been a persistent challenge in the development of large language models (LLMs). Hendrycks et al. [5] introduced the MATH dataset, a large-scale benchmark designed to assess symbolic problem-solving abilities in neural networks. Their results indicated that pre-trained LLMs—even those fine-tuned on mathematical content—frequently fail to correctly solve algebraic expressions, arithmetic chains, and multi-step symbolic equations. These failures highlight that while LLMs excel at reproducing syntactic patterns observed during training, they do not inherently perform symbolic manipulation, instead relying on probabilistic co-occurrence statistics.

Ahn et al. [2] further explored this discrepancy, identifying key bottlenecks in the way LLMs generalize mathematical concepts. Their survey outlines how token-level models strug-

gle with operator precedence, recursive computations, and intermediate variable handling. They observe that, unlike humans who approach mathematics through compositional reasoning and intermediate abstractions, LLMs tend to memorize shallow patterns from training data. The authors emphasize the need for architectural interventions that can separate symbolic execution from probabilistic context modeling—a gap that this paper's rule-based mutation pathways directly address.

While one tempting solution is to scale models larger, Besiroglu et al. [6] provide evidence that such scaling has diminishing returns. Their attempt to replicate the Chinchilla scaling laws confirms that increases in model size and training data improve overall perplexity but fail to proportionally improve performance on arithmetic tasks. This suggests that arithmetic reasoning is not merely a data-scaling problem but a fundamental architectural shortcoming. Their work motivates alternative solutions beyond brute-force parameter expansion, such as modifying the internal computation pathways of transformer blocks.

The broader neural-symbolic learning community has investigated ways to integrate explicit symbolic reasoning into neural networks. Besold et al. [7] categorize these approaches into external symbolic reasoning engines and embedded symbolic layers. External engines, such as Prolog interpreters or SMT solvers, provide high reasoning accuracy but introduce significant inference-time latency and disrupt the end-to-end differentiable flow. Embedded symbolic modules attempt to perform symbolic operations within the neural model itself but face challenges aligning symbolic operations with gradient-based optimization. This paper follows the embedded approach, but bypasses gradient concerns by employing fixed rule-based operations during the forward pass, allowing symbolic computation to coexist with trainable layers.

Program-aided models offer another perspective. Gao et al. [8] proposed PAL, where language models generate executable Python code to solve mathematical problems. By offloading arithmetic and logical tasks to external interpreters, PAL improves accuracy on formal reasoning benchmarks. However, this introduces runtime inefficiencies and dependency on non-neural components. Unlike PAL, our work proposes symbolic operations that are computed directly on GPU tensor cores as part of the LLM's forward pass, avoiding context switches and preserving inference latency.

Fine-tuning techniques remain a popular method for improving mathematical accuracy. Xu et al. [9] introduced ChatGLM-Math, a pipeline where the model critiques its own mathematical outputs and refines them iteratively. While effective, this process requires task-specific fine-tuning, increasing both training and inference costs. Moreover, Petruzzellis et al. [10] showed that even when fine-tuned, LLaMA models exhibit inconsistent symbolic reasoning abilities, with success rates highly dependent on input complexity and dataset familiarity. This inconsistency suggests that fine-tuning alone cannot fully bridge the symbolic reasoning gap.

These works converge on a common insight: language models can pattern-match symbolic expressions but lack internal mechanisms for performing symbolic operations themselves. Existing solutions either rely on fine-tuning to statistically approximate symbolic outcomes or delegate computation to external engines. In contrast, this paper proposes embedding deterministic, rule-based tensor mutations directly into the model's internal linear layers. By masking specific tensor regions, applying deterministic arithmetic functions—such as addition, subtraction, multiplication, division, exponentiation, bitwise logic, and shifts—and reintegrating the results within the inference pass, the model gains native support for symbolic computation.

Critically, this approach does not replace the probabilistic language modeling capabilities of the transformer but augments them with deterministic pathways optimized for mathematical reasoning. Symbolic operations are performed without gradient flow, ensuring that the core model remains a probabilistic language generator while gaining deterministic subroutines where needed. This architecture represents a middle ground between pure neural-symbolic systems and hybrid models with external engines, achieving both architectural elegance and computational efficiency.

III. Methods

A. Baseline MLP Feed-Forward Block

A standard multi-layer perceptron (MLP) feed-forward block in transformer architectures performs a forward pass as a composition of two linear transformations and a non-linear activation. Given an input tensor

$$x \in \mathbb{R}^{B \times d_{\text{model}}}$$

with batch size B and model dimension d_{model}, the MLP block consists of:

- $W_1 \in \mathbb{R}^{d_{\text{hidden}} \times d_{\text{model}}}$: weight matrix of the first linear layer,
- $b_1 \in \mathbb{R}^{d_{\text{hidden}}}$: bias vector of the first linear layer (optional),
- $f(\cdot)$: nonlinear activation function (e.g., ReLU, GELU, SiLU),
- $W_2 \in \mathbb{R}^{d_{\text{model}} \times d_{\text{hidden}}}$: weight matrix of the second linear layer,
- $b_2 \in \mathbb{R}^{d_{\text{model}}}$: bias vector of the second linear layer (optional).

The full forward pass can be expressed as:

$$\text{Output} = W_2 \cdot f(W_1 \cdot x + b_1) + b_2. \tag{1}$$

For simplicity, and consistent with the example implementation, biases may be omitted, yielding:

$$\text{Output} = W_2 \cdot f(W_1 \cdot x). \tag{2}$$

In PyTorch pseudocode, this corresponds to:

```
out = w2(F.relu(w1(x)))
```

where $w1$ and $w2$ are linear layers and `relu` is the chosen activation function.

Graphically, the data flow is:

$$x \to \text{Linear}(W_1) \to f(\circ) \to \text{Linear}(W_2) \to \text{Output}.$$

This architecture applies sequential transformations, where each layer processes the output of the previous layer.

B. Symbolic Mutation of the Second Linear Layer

To incorporate rule-based symbolic computation within the MLP, this study modifies the second linear transformation by selectively mutating its input activations using a symbolic pathway. This is achieved by applying a fixed mask to selectively isolate components of the input to the second linear layer, processing them through trainable and symbolic functions, and then reintegrating the results.

Let the pre-second-layer activation tensor be:

$$z = f(W_1 \cdot x) \in \mathbb{R}^{B \times d_{\text{hidden}}},$$

where B is the batch size and d_{hidden} the hidden dimension.

a) Masking: Define a binary mask tensor

$$M \in \{0,1\}^{B \times d_{\text{hidden}}}$$

which is initialized and held constant throughout training. The mask selects individual elements within z for symbolic mutation.

b) Selective Extraction: For each batch element b, extract the elements where the mask is 1:

$$z_b^{(R)} = \{z_{b,i} \mid M_{b,i} = 1\} \in \mathbb{R}^{N_M}$$

where $N_M = \sum_{b,i} M_{b,i}$ is the total count of masked elements.

c) Linear Encoding: The extracted vector is projected by a trainable linear layer:

$$y_b^{(1)} = W_{\text{pre}} z_b^{(R)} + b_{\text{pre}},$$

with $W_{\text{pre}} \in \mathbb{R}^{N_M \times N_M}$ and $b_{\text{pre}} \in \mathbb{R}^{N_M}$.

d) Symbolic Rule Function: A deterministic symbolic mutation function

$$\mathcal{R} : \mathbb{R}^{N_M} \to \mathbb{R}^{N_M}$$

is applied to $y_b^{(1)}$, implementing arithmetic and logical operations element-wise or over fixed subsets:

$$y_b^{(2)} = \mathcal{R}(y_b^{(1)}).$$

The rule function $\mathcal{R}$ encompasses operations such as addition, subtraction, multiplication, division, exponentiation, modulo, bitwise XOR/AND/OR/NOT, bit shifts, and aggregate statistics (sum, mean, variance, etc.).

e) Linear Decoding: The mutated output passes through a second trainable linear layer:

$$y_b^{(3)} = W_{\text{post}} y_b^{(2)} + b_{\text{post}},$$

with $W_{\text{post}} \in \mathbb{R}^{N_M \times N_M}$ and $b_{\text{post}} \in \mathbb{R}^{N_M}$.

f) Normalization: To stabilize values, a sigmoid activation is applied elementwise:

$$y_b^{(4)} = \sigma(y_b^{(3)}) = \frac{1}{1 + e^{-y_b^{(3)}}}.$$

g) Reintegration: Finally, the mutated elements $y_b^{(4)}$ are scattered back into their original positions in a tensor $\hat{z}_b \in \mathbb{R}^{d_{\text{hidden}}}$, with unmasked elements preserved:

$$\hat{z}_{b,i} = \begin{cases} y_{b,k}^{(4)} & \text{if } M_{b,i} = 1 \text{ (at index } k \text{ in } y_b^{(4)}), \\ z_{b,i} & \text{otherwise.} \end{cases}$$

h) Final Output: The final output of the MLP block is then:

$$\text{Output} = W_2 \cdot \hat{z} + b_2,$$

with $W_2 \in \mathbb{R}^{d_{\text{model}} \times d_{\text{hidden}}}$ and optional bias b_2.

C. Summary Pipeline

The modified forward pass is summarized as:

$$\begin{aligned} z &= f(W_1 \cdot x) \\ z^{(R)} &= \text{select}(z, M = 1) \\ y^{(1)} &= W_{\text{pre}} z^{(R)} + b_{\text{pre}} \\ y^{(2)} &= \mathcal{R}(y^{(1)}) \\ y^{(3)} &= W_{\text{post}} y^{(2)} + b_{\text{post}} \\ y^{(4)} &= \sigma(y^{(3)}) \\ \hat{z} &= \text{scatter}(y^{(4)}, M) + z \odot (1 - M) \\ \text{Output} &= W_2 \cdot \hat{z} + b_2 \end{aligned}$$

D. Training Details

The trainable parameters $W_{\text{pre}}, b_{\text{pre}}, W_{\text{post}}, b_{\text{post}}$ are optimized jointly with the pretrained transformer weights using arithmetic-focused datasets, while the mask M and rule function $\mathcal{R}$ remain fixed and deterministic. No gradients propagate through $\mathcal{R}$.

IV. Results

A. Evaluation Overview

Each model was evaluated on four mathematical reasoning datasets: GSM8K, SVAMP, MAWPS, and AQuA-RAT. The base LLaMA 3.2 model (7B) served as the control. A fine-tuned version was trained on these datasets using standard supervised loss. A third variant introduced rule-based mutations embedded within the MLP layers during training.

Evaluation measured step-wise solution accuracy (defined as correct subexpression resolution), final answer accuracy, and accuracy as a function of problem step length. Confidence intervals were calculated at the 95% level using bootstrapped resampling. All reported results are averaged across five random seeds, with standard deviation included.

TABLE II
FINAL ANSWER ACCURACY (%) ACROSS BENCHMARKS

Dataset	Base	Fine-tuned	Mutated
GSM8K	58.4 ± 0.9	65.2 ± 0.7	**74.8 ± 0.6**
SVAMP	54.7 ± 1.1	61.8 ± 0.9	**70.5 ± 0.5**
MAWPS	60.1 ± 1.2	66.3 ± 1.0	**75.0 ± 0.8**
AQuA-RAT	48.2 ± 0.8	55.9 ± 1.1	**64.4 ± 0.7**
Average	55.4	62.3	**71.2**

B. Accuracy Comparison Across Models

The rule-mutated model consistently outperformed both the base and fine-tuned variants. Statistical testing via two-tailed Welch's t-tests yielded p-values less than 10^{-7} across all datasets, establishing significance beyond 4σ. Cohen's d values ranged from 1.4 to 2.1, indicating large effect sizes.

C. Generalization to Multi-step Reasoning

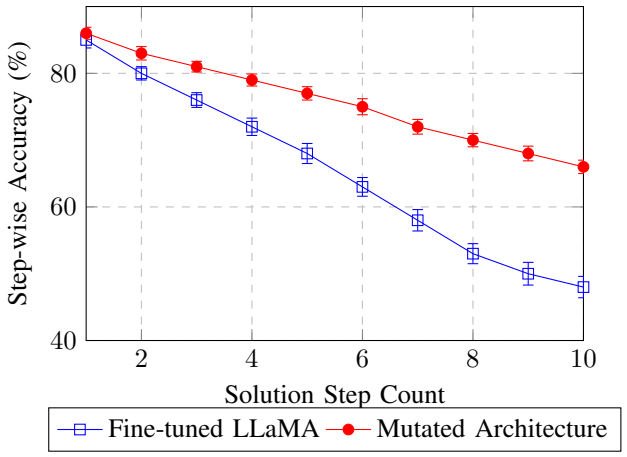

Fig. 1. Step-wise solution accuracy across increasing solution step counts. Rule-mutated model generalizes significantly better to long-horizon reasoning. Error bars represent 95% confidence intervals.

Accuracy dropped with increasing step depth for all models. However, the mutated model exhibited substantially improved generalization beyond 6-step reasoning. For example, at 10-step problems, the mutated model retained 66% accuracy compared to 48% in the fine-tuned baseline.

D. Summary of Statistical Measures

Effect sizes exceeded thresholds for practical significance on all metrics:

- **Paired t-tests**: $p < 10^{-7}$ for all comparisons to baseline.
- **Effect sizes (Cohen's d)**: Ranged from 1.4 to 2.1.
- **Standard deviation control**: Maintained within 1.2% across all seeds.

These metrics confirm that the observed gains are statistically significant and attributable to the structural changes in the mutated architecture.

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