Calculus III

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Contents

Chapter 1

Lab 7

1.1 Work

Question 1

$$\vec{r} \in \mathbb{R} \therefore t \geq 0 \because e^{\sqrt{t}} \notin \mathbb{R}\ t | t < 0$$

This corresponds with answer choice D

Question 2

$$\forall t > 4, \quad \vec{r} \cdot \hat{j} \notin \mathbb{R}$$

$\ln(t-1)$ is not defined $\forall t \leq 1$

This corresponds with answer choice D

Question 3

$$\vec{r} \cdot \hat{j} \in [-4, 4] \quad \wedge \quad \vec{r} \cdot \hat{i} \in [-3, 3]$$

$$\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \langle -3\sin t, 4\cos t \rangle$$

$$\frac{\mathrm{d}\vec{r}}{\mathrm{d}t}_{t=\frac{\pi}{2}} = \langle -3, 0 \rangle \quad \vec{r}(0) = \langle 3, 0 \rangle \qquad dotted$$

This corresponds with answer choice B

Question 4

This can be directly evaluated to:

$$\langle 28, -49 \rangle$$

This corresponds with answer choice D

Question 5

$\vec{r} \cdot \hat{j}$ is not defined for $t = 1$ but the limit as $t \to 6$ can be evaluated without affecting the proceess

$\vec{r} \cdot \hat{i}$ is not defined for $t = 1$ but the limit as $t \to 6$ can be evaluated without affecting the proceess

Using directevaluation:

$$\langle \frac{5}{35}, -\frac{36 + 12 - 3}{5} \rangle = \langle \tfrac{1}{7}, -9 \rangle$$

This corresponds with answer choice B

Question 6

$$\implies \langle 0, -6 \rangle$$

This corresponds with answer choice B

Question 7

$$e^{-\ln 6} = 6^{-1} = \tfrac{1}{6}$$

$$\implies \lim_{t \to \ln 6} \langle 6e^{-t}, 3e^{-t} \rangle = \langle 1, \tfrac{1}{2} \rangle$$

This corresponds witha nswer choice B

Question 8

$$\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \langle -14t, \tfrac{1}{3}t^2 \rangle$$

This corresponds with answer choice C

Question 9

$$\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \langle -\csc^2 t, -\cot t \csc t \rangle$$

This corresponds with answer choice A

Question 10

$$\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \langle 8te^{t^2}, -3, 2t \rangle$$

This corresponds with answer choice B

Question 11

$$\frac{d\vec{r}}{dt} = \langle 18\frac{1}{6t}, 6t^2 \rangle$$
$$\frac{d^2\vec{r}}{dt^2} = \langle \frac{-108}{36t^2}, 12t \rangle \langle \frac{-3}{t^2}, 12t \rangle$$

This corresponds with answer choice C

Question 12

$$\frac{d\vec{r}}{dt} = \langle 15t^4, -60t^4, 20t^4 \rangle$$
$$\| \frac{d\vec{r}}{dt} \| = \sqrt{15^2 \cdot t^8 + 60^2 \cdot t^8 + 20^2 \cdot t^8}$$
$$= t^4\sqrt{15^2 + 60^2 + 20^2} = 65t^4$$
$$\hat{T} = \langle \tfrac{15}{65}, \tfrac{-60}{65}, \tfrac{20}{65} \rangle$$

This corresponds with answer choice B

Question 13

$$\frac{d\vec{r}}{dt} = \langle 36\sin^2(2t)\cos(2t), -36\cos^2(2t)\sin(2t) \rangle$$
$$\| \frac{d\vec{r}}{dt} \| = \sqrt{36^2\sin^4(2t)\cos^2(2t) + 36^2\cos^4(2t)\sin^2(2t)}$$
$$= \sqrt{(36^2\sin^2(2t)\cos^2(2t)) \cdot (\sin^2(2t) + cos^2(2t))}$$
$$= 36\sin(2t)\cos(2t)$$
$$= 18\sin(4t)$$