Calculus III
Homework # 1

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Chapter 1

Lab 2 - 13.1 Apps

1.1 Work

Question 1

Let $\vec{T}_1$ represent the tension of the leftmost cable, while $\vec{T}_2$ encodes the tension force experienced by the rightmost cable. Our coordinate system will originate at the intersection of the two cables.

$$\begin{aligned}
\vec{T}_1 &:= \langle \|\vec{T}_1\|; 135°\rangle \\
\vec{T}_2 &:= \langle \|\vec{T}_2\||; 15°\rangle \\
500 &= \|\vec{T}_1\| \sin 135° + \|\vec{T}_2\| \sin 15 \\
0 &= \|\vec{T}_1\| \cos 135° + \|\vec{T}_2\| \cos 15 \\
\implies 500 &= \|\vec{T}_1\| \frac{\sqrt{2}}{2} + \|\vec{T}_2\| \cdot 0.258819045103 \\
\implies 0 &= \|\vec{T}_1\| \frac{-\sqrt{2}}{2} + \|\vec{T}_2\| \cdot 0.965925826289 \\
&\text{Solving the system numerically yields:} \\
\|\vec{T}_1\| &\Rightarrow 557.677\,\text{lb} \\
\|\vec{T}_2\| &\Rightarrow 408.248\,\text{lb} \\
&\text{This corresponds to answer choice A}
\end{aligned}$$

Question 2

Let $\vec{T}$ represent the tension in the cable, and $\vec{C}$ represent the compression force experienced by the boom in neutralizing the horizontal component of $\vec{T}$.

$$\begin{aligned}
\vec{T} &:= \langle \|\vec{T}\|; 142°\rangle \\
\|\vec{T}\| \sin 142° &= 450\,\text{lb} \\
\|\vec{T}\| &= \frac{450\,\text{lb}}{\sin 142°} \\
\|\vec{T}\| &\Rightarrow 730.921\,\text{lb} \\
\|\vec{C}\| &= |(\|\vec{T}\| \cos 142°)| \Rightarrow 575.973\,\text{lb} \\
&\text{This corresponds to answer choice C}
\end{aligned}$$

Question 3

Let the x'-axis of the new coordinate system be oriented along the unit vector $\hat{u} = \hat{T}_1$:

$$\vec{T}_1 := \langle 3600, 0\rangle$$
$$\vec{T}_2 := \langle 1800; 45^\circ\rangle$$
$$\vec{T}_1 + \vec{T}_2 = \langle 3600 + 1800\cos 45^\circ, 1800\sin 45^\circ\rangle = \langle 3600 + 1800\tfrac{\sqrt{2}}{2}, 1800\tfrac{\sqrt{2}}{2}\rangle$$
$$\|\vec{T}_1 + \vec{T}_2\| \Rightarrow 5036.278\,\text{lb}$$

This corresponds with answer choice D

Question 4

Let the force vector be $\vec{F}$.

$$\vec{F} := 8\cdot\cos 30^\circ\hat{\imath} + 8\cdot\sin 30^\circ\hat{\jmath} = 8\cdot\tfrac{\sqrt{3}}{2}\hat{\imath} + 8\cdot\tfrac{1}{2}\hat{\jmath} = 4\cdot\sqrt{3}\hat{\imath} + 4\hat{\jmath}$$

This corresponds with answer choice A

Question 5

Let the velocity vector be $\vec{v}_0$.

$$\vec{v}_0 := 5\cdot\cos 56^\circ\hat{\imath} + 5\cdot\sin 56^\circ\hat{\jmath} = 2.795\hat{\imath} + 4.145\hat{\jmath}$$

This corresponds with answer choice A

Question 6

Let the velocity vector be $\vec{v}_0$.

$$\vec{v}_0 := \langle 817; 140^\circ\rangle = \langle 817\cos 140^\circ, 817\sin 140^\circ\rangle \Rightarrow \langle -625.858, 525.157\rangle$$

This corresponds with answer choice A

Question 7

Let the wind's parameters be encoded in the vector $\vec{W}$.

$$\vec{W} := \langle 5, 14\rangle$$
$$\implies \theta = \arctan\tfrac{14}{5} \Rightarrow 70.346^\circ$$

This corresponds with answer choice C

Question 8

Let the boat's target velocity vector be $\vec{v}$.

$$\begin{aligned} \vec{v} &:= \langle 0, 31 \rangle - \langle -6, 0 \rangle \\ \vec{v} &= \langle 6, 31 \rangle \\ \implies \theta &= \arctan \tfrac{31}{6} \Rightarrow 79.04593^\circ \end{aligned}$$

This corresponds with answer choice C, when expressed as a bearing East of North

Chapter 2

Lab 3 - 13.2

2.1 Work

Question 1

Let P be the plane in question.

$$P = \{(x, y, z) \in \mathbb{R}^3 : x = 1\}$$

Question 2

$$\text{dist}(P_1, P_2) = \sqrt{(5-1)^2 + (-6+1)^2 + (-5+2)^2} = \sqrt{4^2 + 5^2 + 3^2} = \sqrt{16 + 25 + 9} = \sqrt{50} = 5\sqrt{2}$$

Question 3

$$5^2 = (x+8)^2 + (y-10)^2 + z^2 = x^2 + 8^2 + 16x + y^2 + 100 - 20y + z^2$$

This corresponds with answer choice A

Question 4

Let C be the center point of the sphere, and r be the radius.

$$\begin{aligned} x^2 + y^2 + z^2 - 18x - 10y - 6z &= -15 \\ \implies x^2 - 18x + (\tfrac{18}{2})^2 + y^2 - 10y + (\tfrac{10}{2})^2 + z^2 - 6z + (\tfrac{6}{2})^2 &= -15 + (\tfrac{18}{2})^2 + (\tfrac{10}{2})^2 + (\tfrac{6}{2})^2 \\ \implies (x-9)^2 + (y-5)^2 + (z-3)^2 &= -15 + 81 + 25 + 9 = 10 + 81 + 9 = 100 = 10^2 \\ \therefore C &= (9, 5, 3) \quad r = 10 \end{aligned}$$

Question 5

The set $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 > 1\}$ can be described as the set of all real points outside of a sphere with radius one centered at the origin, non-inclusive of the boundary where $x^2 + y^2 + z^2 = 1$. This corresponds with answer choice C.

Question 6

$$\vec{v} = \vec{PQ} = Q - P \quad Q = (4, 3, -3) \quad P = (-1, -3, 0)$$
$$\vec{v} = \langle 4 + 1, 3 + 3, -3 \rangle = \langle 5, 6, -3 \rangle = 5\hat{\imath} + 6\hat{\jmath} - 3\hat{k}$$
$$\text{This corresponds with answer choice A}$$

Question 7

$$\vec{v} = \vec{AB} = B - A \quad B = (-2, -13, -2) \quad A = (-7, -6, -5)$$
$$\vec{v} = \langle -2 + 7, -13 + 6, -2 + 5 \rangle = \langle 5, -7, 3 \rangle = 5\hat{\imath} - 7\hat{\jmath} + 3\hat{k}$$
$$\text{This corresponds with answer choice B}$$

Question 8

$$M \coloneqq \text{midpoint}(A, B) = \left(\frac{3+5}{2}, \frac{5+2}{2}, \frac{5+4}{2}\right) = (4, 3.5, 4.5)$$
$$\vec{v} = M - C = \langle 4 - 1, 3.5 - 1, 4.5 - 1 \rangle = \langle 3, 2.5, 3.5 \rangle$$
$$\text{This corresponds with answer choice B}$$

Question 9

$$\vec{v} \coloneqq 2\vec{u} - 6\vec{v} \quad \vec{u} = \langle 1, 1, 0 \rangle \quad \vec{v} = \langle 3, 0, 1 \rangle$$
$$\vec{v} = \langle 2, 2, 0 \rangle - \langle 18, 0, 6 \rangle = \langle -16, 2, -6 \rangle = -16\hat{\imath} + 2\hat{\jmath} - 6\hat{k}$$
$$\text{This corresponds with answer choice B}$$

Question 10

The provided vector corresponds with answer choice D since:

$$5\hat{\imath} + 10\hat{\jmath} + 10\hat{k} = 15\left(\tfrac{1}{3}\hat{\imath} + \tfrac{2}{3}\hat{\jmath} + \tfrac{2}{3}\hat{k}\right)$$
$$\implies 5\hat{\imath} + 10\hat{\jmath} + 10\hat{k} = \tfrac{15}{3}\hat{\imath} + \tfrac{30}{3}\hat{\jmath} + \tfrac{30}{3}\hat{k}$$
$$\implies \cancel{5\hat{\imath} + 10\hat{\jmath} + 10\hat{k}} = \cancel{5\hat{\imath} + 10\hat{\jmath} + 10\hat{k}}$$
$$\implies 0 = 0$$

Question 11

Let $\vec{v} = \langle -1, 6, 0 \rangle$

$$\|\vec{v}\| = \sqrt{1^2 + 6^2 + 0^2} = \sqrt{1 + 36} = \sqrt{37}$$

Chapter 3

Lab 4 - 13.3, 13.4

3.1 Work

Question 1

$$\vec{u} := \langle -5, 7 \rangle \quad \vec{v} := \langle 1, 6 \rangle \quad \vec{w} := \langle -11, 2 \rangle$$
$$\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot (\langle 1 - 11, 6 + 2 \rangle) = \vec{u} \cdot \langle -10, 8 \rangle$$
$$\implies \vec{u} \cdot (\vec{v} + \vec{w}) = -5(-10) + 7(8) = 50 + 56 = 106$$

Question 2

$$\vec{r} := \langle 7, -1, -3 \rangle \quad \vec{v} := \langle 2, 6, 6 \rangle \quad \vec{w} := \langle 7, 4, 7 \rangle$$
$$\vec{w} \cdot (\vec{v} + \vec{r}) = \vec{w} \cdot \langle 9, 5, 3 \rangle = 63 + 20 + 21 = 104$$

Question 3

Let o be the work done by the system.

$$o = \langle 158; 180^\circ - 15^\circ \rangle \cdot \langle -6, 0 \rangle = \langle 158 \cos 180^\circ - 15^\circ, 158 \sin 180^\circ - 15^\circ \rangle \cdot \langle -6, 0 \rangle$$
$$o = \langle -152.616280554, 40.8934091262 \rangle \cdot \langle -6, 0 \rangle = 915.697683324$$

This corresponds with answer choice B

Question 4

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \cdot \|\vec{v}\| \cos \theta$$
$$\implies \langle 1, -1 \rangle \cdot \langle 4, 5 \rangle = \sqrt{2}\sqrt{4^2 + 5^2} \cos \theta = 4 - 5 = -1 = \sqrt{2}\sqrt{25 + 16} \cos \theta$$
$$\implies \theta = \arccos \frac{-1}{\sqrt{2}\sqrt{41}} = 96.3401917459^\circ$$

This corresponds with answer choice D

Question 5

$$\begin{aligned}
\langle 4,3\rangle \cdot \langle 20,-8\rangle &= 80-24 \neq 0\\
\langle 15,-6\rangle \cdot \langle 20,-8\rangle &= 300+48 \neq 0\\
\langle 20,4\rangle \cdot \langle 20,-8\rangle &= 400-32 \neq 0\\
\because \langle -10,-25\rangle \cdot \langle 20,-8\rangle &= -200+200 = 0\\
\implies \text{Answer choice D is correct.}
\end{aligned}$$

Question 6

If $\vec{v} \cdot \vec{w} = 0$:

$$\begin{aligned}
\langle 1,-x\rangle \cdot \langle -4,-3\rangle &= -4+3x = 0\\
\implies 3x = 4 \quad \therefore x &= \tfrac{4}{3}
\end{aligned}$$

Definition 3.1.1: Scalar component of a projection

$$\text{scal}_{\vec{b}}(\vec{a}) = \frac{\vec{a} \cdot \vec{b}}{||\vec{b}||}$$

Definition 3.1.2: Vector projection

$$\text{proj}_{\vec{b}}(\vec{a}) = \left(\frac{\vec{a} \cdot \vec{b}}{||\vec{b}||}\right)\frac{\vec{b}}{||\vec{b}||}$$

Question 7

$$\begin{aligned}
\vec{v} = \langle 2,3\rangle \quad \vec{w} &= \langle 8,-6\rangle\\
\text{proj}_{\vec{w}}\vec{v} = \frac{\vec{v} \cdot \vec{w}}{||\vec{w}||}\frac{\vec{w}}{||\vec{w}||} &= \frac{16-18}{\sqrt{8^2+6^2}}\frac{\langle 8,-6\rangle}{\sqrt{6^2+8^2}}\\
\implies \text{proj}_{\vec{w}}\vec{v} = \frac{-2}{10} \cdot \langle \tfrac{8}{10}, \tfrac{-6}{10}\rangle = \tfrac{-2}{10} \cdot \tfrac{2}{10} \cdot \langle 4,-3\rangle = \tfrac{-4}{100}\langle 4,-3\rangle &= \tfrac{-1}{25}\langle 4,-3\rangle\\
\text{This corresponds with answer choice C}
\end{aligned}$$

Question 8

$$\begin{aligned}
\vec{u} = \langle 4,5\rangle \quad \vec{v} &= \langle 1,1\rangle\\
\text{scal}_{\vec{v}}\vec{u} &= \frac{\langle 4,5\rangle \cdot \langle 1,1\rangle}{\sqrt{2}}\\
\implies \text{scal}_{\vec{v}}\vec{u} = \frac{4+5}{\sqrt{2}} &= \tfrac{9}{\sqrt{2}}\\
\text{This corresponds with answer choice A}
\end{aligned}$$

Question 9

$$\begin{aligned}
\vec{u} = \langle 0, 7, -2\rangle \quad \vec{v} = \langle 4, -6, -7\rangle \\
\vec{u}\cdot\vec{v} = \|\vec{u}\|\cdot\|\vec{v}\|\cos\theta = 0 - 42 + 14 = -28 = \sqrt{7^2+2^2}\sqrt{4^2+6^2+7^2}\cos\theta \\
\theta = \arccos\frac{-28}{\sqrt{53}\sqrt{101}} = 1.9635142449 \text{ rad} \\
\text{This corresponds with answer choice B}
\end{aligned}$$

Question 10

$$\vec{u} = \langle -5, -3, 4\rangle, \quad \vec{v} = \langle 5, -4, 2\rangle, \quad \vec{w} = \langle 10, -5, -8\rangle$$

$$\begin{aligned}
\vec{u}\times\vec{v} &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -5 & -3 & 4 \\ 5 & -4 & 2 \end{vmatrix} \\
&= \hat{i}((-3)(2) - (4)(-4)) - \hat{j}((-5)(2) - (4)(5)) + \hat{k}((-5)(-4) - (-3)(5)) \\
&= \hat{i}(-6+16) - \hat{j}(-10-20) + \hat{k}(20+15) \\
&= \langle 10, 30, 35\rangle \\
\vec{w}\cdot(\vec{u}\times\vec{v}) &= \langle 10, -5, -8\rangle \cdot \langle 10, 30, 35\rangle \\
&= 10\cdot 10 + (-5)\cdot 30 + (-8)\cdot 35 \\
&= 100 - 150 - 280 \\
&= -330
\end{aligned}$$

This corresponds with answer choice B

Definition 3.1.3: Area of a Triangle using a Cross Product

$$\text{Area}_{\triangle ABC} = \frac{1}{2}\|\vec{B} - \vec{A}\times\vec{C} - \vec{A}\|$$

Definition 3.1.4: Area of a Parallelogram using a Cross Product

$$\text{Area}_{\text{parallelogram}} = \|\vec{u}\times\vec{v}\|$$

Question 11

In order to aptly encode the direction and magnitude of the side of the triangle let $\vec{s}_1, \vec{s}_2$ originate from P.

$$\vec{s}_1 = \langle 6-1, 6-1, -3-1\rangle = \langle 5, 5, -4\rangle \quad \vec{s}_2 = \langle 10-1, 4-1, 2-1\rangle = \langle 9, 3, 1\rangle$$

$$A = \tfrac{1}{2}\|\vec{s}_1 \times \vec{s}_2\| = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 5 & -4 \\ 9 & 3 & 1 \end{vmatrix}$$

$$\implies \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 5 & 5 & -4 \\ 9 & 3 & 1 \end{vmatrix} = \hat{i}(5+12) - \hat{j}(5+36) + \hat{k}(15-45) = \langle 17, -41, -30\rangle$$

$$\implies \tfrac{1}{2}\|\vec{s}_1 \times \vec{s}_2\| = \tfrac{1}{2} \cdot \sqrt{17^2 + 41^2 + 30^2} = \tfrac{1}{2}\sqrt{2870}$$

This corresponds with answer choice D

Question 12

$$\|\vec{F} \times \vec{PQ}\| = \|\vec{\tau}\| = \|\vec{PQ}\| \cdot \|\vec{F}\| \sin 45^\circ = \tfrac{9}{12} \cdot 5\frac{\sqrt{2}}{2} = \frac{45\sqrt{2}}{24} = \tfrac{15}{8}\sqrt{2} \text{ ft} \cdot \text{lb}$$

This corresponds with answer choice D

Chapter 4

Lab 5 - 13.5

4.1 Work

Question 1

$$\vec{r} = \langle 0, 1, 0 \rangle + \langle 3, 0, -1 \rangle t$$
breaking it down into component form:
$$x = 3t, \quad y = 1, \quad z = -t$$
This corresponds with answer choice C

Question 2

$$\vec{r} = \langle -6, 5, -5 \rangle t + \langle 5, -1, -5 \rangle$$
This corresponds with answer choice C

Question 3

$$\vec{v} = P_2 - P_1 = \langle 3, 0, 4 \rangle \quad \vec{r} = \langle 3, 0, 4 \rangle t + \langle -3, 7, 3 \rangle \vee \vec{r} = \langle 3, 0, 4 \rangle t + \langle 0, 7, 7 \rangle$$
This corresponds with answer choice B

Question 4

$$\vec{u} = \langle 6, 4, 4 \rangle \quad \vec{v} = \langle -7, -6, -4 \rangle$$
$$\vec{v}_1 := \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 4 & 4 \\ -7 & -6 & -4 \end{vmatrix} = \hat{i}(-16 + 24) - \hat{j}(-24 + 28) + \hat{k}(-36 + 28) = \langle 8, -4, -8 \rangle$$
$$\vec{r} = \vec{v}_1 t + \langle -4, -7, 4 \rangle$$
This corresponds with answer choice D

Question 5

$$\vec{v}_1 = \langle 3, -3, -1 \rangle \quad \vec{v}_2 = \langle 4, -2, -5 \rangle$$
$$\nexists k | k \in \mathbb{R} \land k \cdot \vec{v}_1 = \vec{v}_2$$
$$\therefore l_1 \nparallel l_2$$
$$1 + 3t = 3 + 4s$$
$$3 - 3t = -1 - 2s$$
$$z = -t = 3 - 5s$$

$$s = \frac{1}{4}(3t - 2)$$
$$3 - 3t = -1 - 2 \cdot \frac{1}{4}(3t - 2)$$
$$3 - 3t = -1 - \frac{1}{2}(3t - 2)$$
$$3 - 3t = -\frac{3}{2}t$$
$$3 = \frac{3}{2}t$$
$$t = 2$$
$$s = \frac{1}{4}(3 \cdot 2 - 2) = 1$$

$$x = 1 + 3(2) = 7$$
$$y = 3 - 3(2) = -3$$
$$z = -2$$

$$x = 3 + 4(1) = 7$$
$$y = -1 - 2(1) = -3$$
$$z = 3 - 5(1) = -2$$

$$\therefore (x, y, z) = (7, -3, -2)$$

This corresponds with answer choice D

Question 6

$$\exists k | k \in \mathbb{R} \land k \cdot \vec{v}_1 = \vec{v}_2$$
$$\therefore l_1 \parallel l_2$$
$$3 - t = -6 + 5s$$
$$9 - t = 5s$$
$$9 - 5s = t$$
$$1 + 6(9 - 5s) = 55 - 30s = 1 + 54 - 30s$$
$$55 - 30s = 55 - 30s \forall s \in \mathbb{R}$$
$$4 - 2(9 - 5s) = -14 + 10s = 4 - 18 + 10s$$
$$-14 + 10s = -14 + 10s \forall s \in \mathbb{R}$$

$\therefore$ l_1 and l_2 are identical. This corresponds to answer choice B

Question 7

$$\vec{n}\cdot(\vec{r}-\vec{r}_0)=0 \quad \langle 2,7,6\rangle\cdot(\vec{r}-\langle 4,-3,2\rangle)=0$$
$$\vec{r}|\vec{r}=\langle x,y,z\rangle \implies \langle 2,7,6\rangle\cdot\langle x-4,y+3,z-2\rangle$$
$$2(x-4)+7(y+3)+6(z-2)=2x-8+7y+21+6z-12=0$$
$$2x+7y+6z-8+21-12=0 \quad 2x+7y+6z=-1$$

This corresponds with answer choice C

Question 8

Considering answer choice A:

$$5(-1)-7(8)+58=-5-56+58=-3\neq 3\therefore \cancel{A}$$
$$5(-1)+7(8)-58=-5+56-58=51-58=-7\neq 3\therefore \cancel{B}$$
$$5(-1)+y(8)-58\neq -3\therefore \cancel{C}$$
$$5(-1)-7(8)+58=-5-56+58=-3 \;\therefore\; \text{The answer is D}$$

Question 9

$$-2x+2=-2y$$
$$-2x=2+5z=2$$
$$z_0\coloneqq 0 \quad -2y+5(0)=2 \implies y=-2$$
$$-2x+2(-2)=-2=-2x-4$$
$$-2x=2$$
$$\implies x=-1$$
$$r_0=(-1,-2,0)$$

By parameterizing r_0, only answer choice D has the necessary constant terms.

Question 10

$$x+y=7-z$$
$$7-z=12$$
$$z=-5$$
$$x=t$$
$$t+y=12$$
$$y=12-t$$
$$\therefore x=t \quad y=12-t \quad z=-5$$

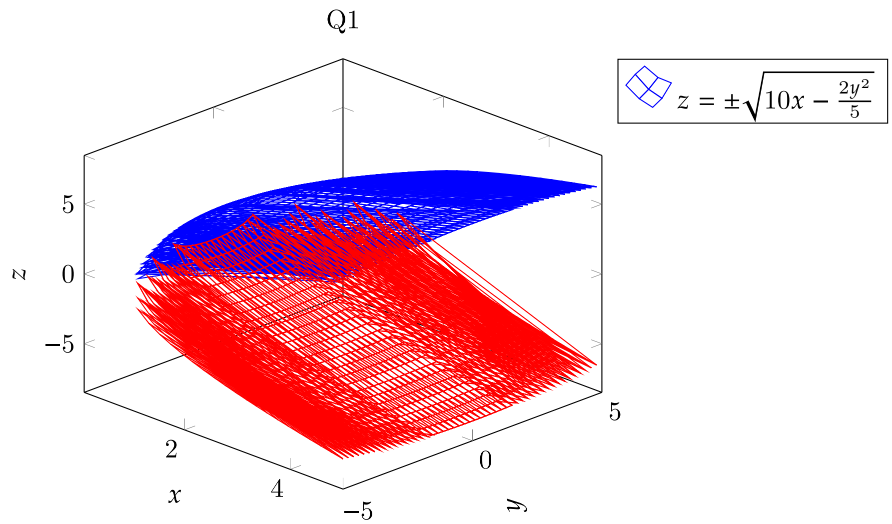
This corresponds with answer choice B. Answer Choice A is the same line with different parametriazation to $x=-t$ instead of $x=t$

Chapter 5

Lab 6 - 13.6

5.1 Work

Question 1



This image corresponds to that of an elliptical paraboloid, corresponding with answer choice A.

Question 2

Since all terms are raised to the second degree, all behave linearly with respect to each other. Therefore, this is not a curved surface. Furthermore, the coefficient of the term containing x is negative. Therefore, the equation represents an elliptical cone along the x-axis. This corresponds with answer choice D.

Question 3

This surface is linear along the y-axis, and has a negative coefficient on the x term. Therefore, the surface is a Hyperbolic paraboloid, corresponding with answer choice A.

Question 4

The surface is linear along the z-axis, guaranteeing it to be a paraboloid. The xz-trace occurs when $y = 0$. The yz-trace occurs when $x = 0$.

$$\begin{aligned} xz\text{-trace:} \quad & 16x^2 - 16(0)^2 = z = 16x^2 \\ yz\text{-trace:} \quad & 16(0)^2 - 16y^2 - z = 0 = 16y^2 + z \end{aligned}$$

This corresponds with answer choice D.

Question 5

Since two terms have a negative coefficient, and all terms are quadratic, the surface must be a hyperboloid of two sheets, leaving only answer choices B and D. A hyperboloid of two sheets only has hyperbolic cross-sections, so B must be the answer.

Question 6

Since the equation only yields circular cross sections, and varies quadratically over $\hat{\imath}, \hat{\jmath}, \hat{k}$, The answer is Figure 1, A.

Question 7

The equation yield a circular cross section for any plane parallel to the yz-plane, and only discontinuous sets for the xy-plane and the xz-plane, the surface must be elliptical $\forall(y, z)$. Only Figure 3 satisfies the conditions, validating answer choice C.