Calculus III
Homework # 1

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Contents

Chapter 1

Lab 2 - 13.1 Apps

1.1 Work

Question 1

Let $\vec{T}_1$ represent the tension of the leftmost cable, while $\vec{T}_2$ encodes the tension force experienced by the rightmost cable. Our coordinate system will originate at the intersection of the two cables.

$$\begin{aligned}
\vec{T}_1 &:= \langle \|\vec{T}_1\|; 135^\circ \rangle \\
\vec{T}_2 &:= \langle \|\vec{T}_2\||; 15^\circ \rangle \\
500 &= \|\vec{T}_1\| \sin 135^\circ + \|\vec{T}_2\| \sin 15 \\
0 &= \|\vec{T}_1\| \cos 135^\circ + \|\vec{T}_2\| \cos 15 \\
\implies 500 &= \|\vec{T}_1\| \frac{\sqrt{2}}{2} + \|\vec{T}_2\| \cdot 0.258819045103 \\
\implies 0 &= \|\vec{T}_1\| \frac{-\sqrt{2}}{2} + \|\vec{T}_2\| \cdot 0.965925826289 \\
&\text{Solving the system numerically yields:} \\
\|\vec{T}_1\| &\Rightarrow 557.677\,\text{lb} \\
\|\vec{T}_2\| &\Rightarrow 408.248\,\text{lb} \\
&\text{This corresponds to answer choice A}
\end{aligned}$$

Question 2

Let $\vec{T}$ represent the tension in the cable, and $\vec{C}$ represent the compression force experienced by the boom in neutralizing the horizontal component of $\vec{T}$.

$$\begin{aligned}
\vec{T} &:= \langle \|\vec{T}\|; 142^\circ \rangle \\
\|\vec{T}\| \sin 142^\circ &= 450\,\text{lb} \\
\|\vec{T}\| &= \frac{450\,\text{lb}}{\sin 142^\circ} \\
\|\vec{T}\| &\Rightarrow 730.921\,\text{lb} \\
\|\vec{C}\| &= |(\|\vec{T}\| \cos 142^\circ)| \Rightarrow 575.973\,\text{lb} \\
&\text{This corresponds to answer choice C}
\end{aligned}$$

Question 3

Let the x'-axis of the new coordinate system be oriented along the unit vector $\hat{u} = \hat{T}_1$:

$$\vec{T}_1 := \langle 3600, 0 \rangle$$
$$\vec{T}_2 := \langle 1800; 45^\circ \rangle$$
$$\vec{T}_1 + \vec{T}_2 = \langle 3600 + 1800 \cos 45^\circ, 1800 \sin 45^\circ \rangle = \langle 3600 + 1800 \tfrac{\sqrt{2}}{2}, 1800 \tfrac{\sqrt{2}}{2} \rangle$$
$$\|\vec{T}_1 + \vec{T}_2\| \Rightarrow 5036.278\,\text{lb}$$

This corresponds with answer choice D

Question 4

Let the force vector be $\vec{F}$.

$$\vec{F} := 8 \cdot \cos 30^\circ \hat{\imath} + 8 \cdot \sin 30^\circ \hat{\jmath} = 8 \cdot \tfrac{\sqrt{3}}{2} \hat{\imath} + 8 \cdot \tfrac{1}{2} \hat{\jmath} = 4 \cdot \sqrt{3} \hat{\imath} + 4 \hat{\jmath}$$

This corresponds with answer choice A

Question 5

Let the velocity vector be $\vec{v}_0$.

$$\vec{v}_0 := 5 \cdot \cos 56^\circ \hat{\imath} + 5 \cdot \sin 56^\circ \hat{\jmath} = 2.795 \hat{\imath} + 4.145 \hat{\jmath}$$

This corresponds with answer choice A

Question 6

Let the velocity vector be $\vec{v}_0$.

$$\vec{v}_0 := \langle 817; 140^\circ \rangle = \langle 817 \cos 140^\circ, 817 \sin 140^\circ \rangle \Rightarrow \langle -625.858, 525.157 \rangle$$

This corresponds with answer choice A

Question 7

Let the wind's parameters be encoded in the vector $\vec{W}$.

$$\vec{W} := \langle 5, 14 \rangle$$
$$\Longrightarrow \quad \theta = \arctan \tfrac{14}{5} \Rightarrow 70.346^\circ$$

This corresponds with answer choice C

Question 8

Let the boat's target velocity vector be $\vec{v}$.

$$\begin{aligned} \vec{v} &:= \langle 0, 31 \rangle - \langle -6, 0 \rangle \\ \vec{v} &= \langle 6, 31 \rangle \\ \implies \theta &= \arctan \tfrac{31}{6} \Rightarrow 79.04593^\circ \end{aligned}$$

This corresponds with answer choice C, when expressed as a bearing East of North

Chapter 2

Lab 3 - 13.2

2.1 Work

Question 1

Let P be the plane in question.

$$P = \{(x, y, z) \in \mathbb{R}^3 : x = 1\}$$

Question 2

$$\text{dist}(P_1, P_2) = \sqrt{(5-1)^2 + (-6+1)^2 + (-5+2)^2} = \sqrt{4^2 + 5^2 + 3^2} = \sqrt{16 + 25 + 9} = \sqrt{50} = 5\sqrt{2}$$

Question 3

$$5^2 = (x+8)^2 + (y-10)^2 + z^2 = x^2 + 8^2 + 16x + y^2 + 100 - 20y + z^2$$

This corresponds with answer choice A

Question 4

Let C be the center point of the sphere, and r be the radius.

$$\begin{aligned} x^2 + y^2 + z^2 - 18x - 10y - 6z &= -15 \\ \implies x^2 - 18x + (\tfrac{18}{2})^2 + y^2 - 10y + (\tfrac{10}{2})^2 + z^2 - 6z + (\tfrac{6}{2})^2 &= -15 + (\tfrac{18}{2})^2 + (\tfrac{10}{2})^2 + (\tfrac{6}{2})^2 \\ \implies (x-9)^2 + (y-5)^2 + (z-3)^2 &= -15 + 81 + 25 + 9 = 10 + 81 + 9 = 100 = 10^2 \\ \therefore C = (9, 5, 3) \quad r &= 10 \end{aligned}$$

Question 5

The set $\{(x, y, z) \in \mathbb{R}^3 : x^2 + y^2 + z^2 > 1\}$ can be described as the set of all real points outside of a sphere with radius one centered at the origin, non-inclusive of the boundary where $x^2 + y^2 + z^2 = 1$. This corresponds with answer choice C.

Question 6

$$\vec{v} = \vec{PQ} = Q - P \quad Q = (4, 3, -3) \quad P = (-1, -3, 0)$$
$$\vec{v} = \langle 4+1, 3+3, -3 \rangle = \langle 5, 6, -3 \rangle = 5\hat{\imath} + 6\hat{\jmath} - 3\hat{k}$$
This corresponds with answer choice A

Question 7

$$\vec{v} = \vec{AB} = B - A \quad B = (-2, -13, -2) \quad A = (-7, -6, -5)$$
$$\vec{v} = \langle -2+7, -13+6, -2+5 \rangle = \langle 5, -7, 3 \rangle = 5\hat{\imath} - 7\hat{\jmath} + 3\hat{k}$$
This corresponds with answer choice B

Question 8

$$M := \text{midpoint}(A, B) = \left(\frac{3+5}{2}, \frac{5+2}{2}, \frac{5+4}{2}\right) = (4, 3.5, 4.5)$$
$$\vec{v} = M - C = \langle 4-1, 3.5-1, 4.5-1 \rangle = \langle 3, 2.5, 3.5 \rangle$$
This corresponds with answer choice B

Question 9

$$\vec{v} := 2\vec{u} - 6\vec{v} \quad \vec{u} = \langle 1, 1, 0 \rangle \quad \vec{v} = \langle 3, 0, 1 \rangle$$
$$\vec{v} = \langle 2, 2, 0 \rangle - \langle 18, 0, 6 \rangle = \langle -16, 2, -6 \rangle = -16\hat{\imath} + 2\hat{\jmath} - 6\hat{k}$$
This corresponds with answer choice B

Question 10

The provided vector corresponds with answer choice D since:

$$\begin{aligned} 5\hat{\imath} + 10\hat{\jmath} + 10\hat{k} &= 15\left(\tfrac{1}{3}\hat{\imath} + \tfrac{2}{3}\hat{\jmath} + \tfrac{2}{3}\hat{k}\right) \\ \implies 5\hat{\imath} + 10\hat{\jmath} + 10\hat{k} &= \tfrac{15}{3}\hat{\imath} + \tfrac{30}{3}\hat{\jmath} + \tfrac{30}{3}\hat{k} \\ \implies \cancel{5\hat{\imath} + 10\hat{\jmath} + 10\hat{k}} &= \cancel{5\hat{\imath} + 10\hat{\jmath} + 10\hat{k}} \\ \implies 0 &= 0 \end{aligned}$$

Question 11

Let $\vec{v} = \langle -1, 6, 0 \rangle$

$$\|\vec{v}\| = \sqrt{1^2 + 6^2 + 0^2} = \sqrt{1+36} = \sqrt{37}$$