Calculus III

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Chapter 1

Lab 7

1.1 Work

Question 1

$$\vec{r} \in \mathbb{R} \therefore t \geq 0 \because e^{\sqrt{t}} \notin \mathbb{R}\ t | t < 0$$

This corresponds with answer choice D

Question 2

$$\forall t > 4, \quad \vec{r} \cdot \hat{j} \notin \mathbb{R}$$

$\ln(t-1)$ is not defined $\forall t \leq 1$

This corresponds with answer choice D

Question 3

$$\vec{r} \cdot \hat{j} \in [-4, 4] \quad \wedge \quad \vec{r} \cdot \hat{i} \in [-3, 3]$$

$$\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \langle -3\sin t, 4\cos t \rangle$$

$$\frac{\mathrm{d}\vec{r}}{\mathrm{d}t}_{t=\frac{\pi}{2}} = \langle -3, 0 \rangle \quad \vec{r}(0) = \langle 3, 0 \rangle \qquad dotted$$

This corresponds with answer choice B

Question 4

This can be directly evaluated to:

$$\langle 28, -49 \rangle$$

This corresponds with answer choice D

Question 5

$\vec{r} \cdot \hat{j}$ is not defined for $t = 1$ but the limit as $t \to 6$ can be evaluated without affecting the proceess

$\vec{r} \cdot \hat{i}$ is not defined for $t = 1$ but the limit as $t \to 6$ can be evaluated without affecting the proceess

Using directevaluation:

$$\langle \frac{5}{35}, -\frac{36 + 12 - 3}{5} \rangle = \langle \tfrac{1}{7}, -9 \rangle$$

This corresponds with answer choice B

Question 6

$$\implies \langle 0, -6 \rangle$$

This corresponds with answer choice B

Question 7

$$e^{-\ln 6} = 6^{-1} = \tfrac{1}{6}$$

$$\implies \lim_{t \to \ln 6} \langle 6e^{-t}, 3e^{-t} \rangle = \langle 1, \tfrac{1}{2} \rangle$$

This corresponds witha nswer choice B

Question 8

$$\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \langle -14t, \tfrac{1}{3}t^2 \rangle$$

This corresponds with answer choice C

Question 9

$$\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \langle -\csc^2 t, -\cot t \csc t \rangle$$

This corresponds with answer choice A

Question 10

$$\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \langle 8te^{t^2}, -3, 2t \rangle$$

This corresponds with answer choice B

Question 11

$$\frac{d\vec{r}}{dt} = \langle 18\frac{1}{6t}, 6t^2 \rangle$$
$$\frac{d^2\vec{r}}{dt^2} = \langle \frac{-108}{36t^2}, 12t \rangle \langle \frac{-3}{t^2}, 12t \rangle$$

This corresponds with answer choice C

Question 12

$$\frac{d\vec{r}}{dt} = \langle 15t^4, -60t^4, 20t^4 \rangle$$
$$\| \frac{d\vec{r}}{dt} \| = \sqrt{15^2 \cdot t^8 + 60^2 \cdot t^8 + 20^2 \cdot t^8}$$
$$= t^4\sqrt{15^2 + 60^2 + 20^2} = 65t^4$$
$$\hat{T} = \langle \tfrac{15}{65}, \tfrac{-60}{65}, \tfrac{20}{65} \rangle$$

This corresponds with answer choice B

Question 13

$$\vec{r}(t) = 6\sin^3(2t)\,\hat{i} + 6\cos^3(2t)\,\hat{j}$$
$$\frac{d\vec{r}}{dt} = \langle 36\sin^2(2t)\cos(2t),\ -36\cos^2(2t)\sin(2t) \rangle$$
$$= 36\sin(2t)\cos(2t)\,(\sin(2t)\,\hat{i} - \cos(2t)\,\hat{j})$$

$$\left\| \frac{d\vec{r}}{dt} \right\| = \sqrt{(36\sin^2(2t)\cos(2t))^2 + (-36\cos^2(2t)\sin(2t))^2}$$
$$= 36\sqrt{\sin^4(2t)\cos^2(2t) + \cos^4(2t)\sin^2(2t)}$$
$$= 36\sqrt{\sin^2(2t)\cos^2(2t)}$$
$$= 36\,|\sin(2t)\cos(2t)|$$
$$= 18\,|\sin(4t)|$$

$$\hat{T}(t) = \frac{\dfrac{d\vec{r}}{dt}}{\left\| \dfrac{d\vec{r}}{dt} \right\|} = \frac{36\sin(2t)\cos(2t)(\sin(2t)\,\hat{i} - \cos(2t)\,\hat{j})}{36|\sin(2t)\cos(2t)|}$$
$$= \sin(2t)\,\hat{i} - \cos(2t)\,\hat{j}$$

This corresponds with answer choice B.

Question 14

Let $\vec{C}$ encode the components of the constant of integration such that $\vec{C} \in \mathbb{R}^3$

$$\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \langle t^4 + 5t^2, 4t \rangle$$

$$\int \mathrm{d}\vec{r} = \int \langle t^4 + 5t^2, 4t \rangle \mathrm{d}t$$

$$\vec{r}(t) = \langle \tfrac{1}{5}t^5 + \tfrac{5}{3}t^3, 2t^2 \rangle + \vec{C}$$

$$\vec{C} = \vec{r}(0) = \langle 5, -6 \rangle$$

This corresponds with answer choice C.

Question 15

$$\implies \int_0^3 \left\langle \frac{2}{\sqrt{1+t}}, -9t^2, \frac{6t}{(1+t^2)^2} \right\rangle \mathrm{d}t$$

$$\implies \int_0^3 \left\langle 2(1+t)^{-\frac{1}{2}}, -9t^2, \frac{6t}{(1+t^2)^2} \right\rangle \mathrm{d}t$$

$$\implies \left[\langle 4\sqrt{t+1}, -3t^3, \frac{-3}{1+t^2} \right]_0^3 = \langle 8-4, -81, \tfrac{-3}{10} + 3 \rangle = \langle 4, -81, \tfrac{27}{10} \rangle$$

This corresponds with answer choice C.

Question 16

$$u \coloneqq t^2 + 1 \; \mathrm{du} = 2t\mathrm{d}t \quad \implies \left\langle \left[8t^3 - 3t \right]_0^1, \int_0^1 \frac{4}{u}\mathrm{d}u, -\int_0^1 \frac{1}{2\sqrt{u}}\mathrm{d}u \right\rangle$$

$$\left[\left\langle 8t^3 - 3t, 4\ln(t^2+1), -\sqrt{t^2+1} \right\rangle \right]_0^1 = \langle 8-3, 4\ln 2 - 0, -\sqrt{2} + 1 \rangle$$

This corresponds with answer choice A.

Chapter 2

Lab 8

2.1 Work

Question 1

$$\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \vec{v} = \langle -14t, \tfrac{1}{7}t^2 \rangle$$

This corresponds with answer choice B.

Question 2

$$\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \vec{v} = \langle -3\sin(3t), 5\cos t \rangle$$
$$\frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2} = \vec{a} = \langle -9\cos(3t), -5\sin t \rangle$$

This corresponds with answer choice D.

Question 3

$$\vec{v}(t) = \langle 18t + 4, -15t^2, -2t \rangle$$
$$\vec{v}(3) = \langle 58, -135, -6 \rangle$$

This correspon ds with answer choice B.

Question 4

$$\frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \vec{v} = \langle -2\sin(2t), \frac{7}{t-3}, -\tfrac{1}{3}t^2 \rangle$$
$$\vec{v}(0) = \langle 0, \tfrac{-7}{3}, 0 \rangle$$

This corresponds with answer choice D.

Question 5

$$\frac{d\vec{r}}{dt} = \vec{v} = \langle 8\cos(2t),\ 10\sin(2t),\ -6\csc(2t)\cot(2t)\rangle$$

$$\frac{d^2\vec{r}}{dt^2} = \vec{a} = \langle -16\sin(2t),\ 20\cos(2t),\ 12(\csc(2t)\cot^2(2t) + \csc^3(2t))\rangle$$

$$\vec{a}\left(\tfrac{\pi}{4}\right) = \langle -16\sin\left(\tfrac{\pi}{2}\right),\ 20\cos\left(\tfrac{\pi}{2}\right),\ 12(1\cdot 0^2 + 1^3)\rangle$$

$$= \langle -16,\ 0,\ 12\rangle$$

This corresponds with answer choice B.

Question 6

$$\vec{v}(0) = \langle\sqrt{2}, 0, \sqrt{2}\rangle, \qquad \vec{a}(0) = \langle 0, 0, \tfrac{\pi}{2}\rangle,$$

$$\vec{a}\cdot\vec{v} = 0\cdot\sqrt{2} + 0\cdot 0 + \tfrac{\pi}{2}\cdot\sqrt{2} = \tfrac{\pi\sqrt{2}}{2},$$

$$\|\vec{a}\| = \tfrac{\pi}{2}, \qquad \|\vec{v}\| = \sqrt{(\sqrt{2})^2 + 0^2 + (\sqrt{2})^2} = \sqrt{4} = 2,$$

$$\cos\theta = \frac{\vec{a}\cdot\vec{v}}{\|\vec{a}\|\|\vec{v}\|} = \frac{\frac{\pi\sqrt{2}}{2}}{\frac{\pi}{2}\cdot 2} = \frac{\sqrt{2}}{2},$$

$$\theta = \arccos\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}.$$

Question 7

$$\vec{v}_0 = \langle 75\cos\alpha, 75\sin\alpha\rangle \quad \vec{a} = \langle 0, -32\rangle$$

This corresponds with answer choice A.

Question 8

$$\vec{v}_0 = \langle 800; 34^\circ\rangle \quad \vec{a} = \langle 0, -9.8\rangle$$

$$\vec{r}\cdot\hat{i} = 800\cos 34^\circ t \implies \frac{20000}{800\cdot\cos 34^\circ} = t_h = 30.1554487126 \text{ s}$$

$$\vec{r}(t_h)\cdot\hat{j} = 800\sin 34^\circ 30.1554487126 - \tfrac{1}{2}9.8\cdot 30.1554487126^2$$

$$\vec{r}(t_h)\cdot\hat{j} = 9034.35001026 \geq 0$$

The answer is B.

Question 9

$$\vec{v}(t) = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \langle 585\tfrac{\sqrt{2}}{2}, 585\tfrac{\sqrt{2}}{2} - 9.8t \rangle$$

$$\vec{v}(t) \cdot \hat{j} = 0 \implies t_h = \frac{585\frac{\sqrt{2}}{2}}{9.8} \implies t_h = 42.2099456116$$

$$\vec{r}(t_h) \cdot \hat{j} = 585\tfrac{\sqrt{2}}{2} t_h - \tfrac{1}{2} 9.8 t_h^2 = 8730.22959184 \text{ m}$$

This corresponds with answer choice D.

Question 10

$$12 = \|\vec{v}\| \cos 33° t \quad 0 = \|\vec{v}\| \sin 33° t - \tfrac{1}{2} 9.8 t^2$$

Computationally solving this system of equations: $\|\vec{v}\| = 11.34589$

This corresponds with answewr choice D.

Question 11

$$\vec{r}(t) \cdot \hat{j} = -\tfrac{1}{2} 32 t^2 + 115 \sin 44° t + 6.6$$

$$\vec{r}(4) = 70.1428504111 \text{ ft}$$

This corresponds with answer choice C.

Question 12

$$\vec{r}(t) \cdot \hat{j} = -\tfrac{1}{2} 32 t^2 + 47 \sin 39° t + 6.1$$

$$\vec{r}(t_f) \cdot \hat{j} = 0 \; t_f = 2.03589 \text{ s}$$

$$\vec{r}(t_f) \cdot \hat{i} = 2.03589 \cdot 47 \cos 39° = 74.3626334991 \text{ ft}$$

This corresponds with answer choice C.

Question 13

$$0 = 30 + 39 \sin 26° t - \tfrac{1}{2} 32 t^2$$

$$\implies t = 2.00411 \text{ s}$$

This corresponds with answer choice B.

Question 14

$$x_{max} = \frac{\|\vec{v}_0\|^2 \sin 2\alpha}{g} = 14000 = \frac{380^2 \sin 2\alpha}{9.8}$$

$$\implies \alpha = \tfrac{1}{2} \arcsin \frac{14000 \cdot 9.8}{380^2}$$

This corresponds with answer choice C.